Some Comments on an Analysis of Turbulent Flow in Concentric Annuli

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A theoretical analysis of turbulent flow in concentric annuli has been recently presented by Randhava (*1*). In this work, a simple expression for the velocity profile in the inner wall region is proposed:

$$U_1{}^+ = \frac{K}{\beta} \ln y^+ + A' \qquad (1)$$

The conventional logarithmic profile is accepted for the outer wall region:

$$U_2{}^+ = K \ln y^+ + A \qquad (2)$$

The maximum velocity radius is assumed as given by the Kays and Leung (*2*) correlation. Its knowledge together with the inner and outer radius fully determines the ratio of the wall shear stresses β. Constant A' is computed satisfying the condition

$$U_1{}^+ \,(19.7) = 13.2 \qquad (3)$$

Some comments can be added to the aforementioned paper.

1. The author points out that a variable mixing length constant is in disagreement with the experimental observations and therefore is unacceptable. From Equation (1) it seems that the assumed mixing length constant is β/K and therefore dependent upon r_m and the radius ratio.
2. The dimensional values of inner and outer velocities are not equal at the maximum velocity radius. Once a value of r_m is selected, the velocity profiles are completely determined. The velocity continuity requirement at r_m appears as a further condition which cannot be satisfied.

The latter remark suggests that the maximum velocity radius can be regarded as an unknown and obtained by imposing such a continuity condition. In this way, the following equation can be written:

$$U_{\tau 2}\,[K \ln y^+{}_{m2} + A] = U_{\tau 1}\left[K\left(\frac{U_{\tau 2}}{U_{\tau 1}}\right)^2 \ln\left(\frac{y^+{}_{m1}}{19.7}\right) + 13.2\right] \qquad (4)$$

Predictions of Equation (4) are compared in Table 1 with some experimental data of Brighton and Jones (*3*). The agreement with the experimental results seems to be satisfactory, with the exception of the slight dependence upon the Reynolds number which disagrees with the experimental observations.

TABLE 1.

r_1/r_2	N_{Re}	r_m (cm.) Experimental	r_m (cm.) Equation (4)
0.0625	95,800	3.39	3.16
0.0625	194,000	3.25	3.23
0.0625	327,000	3.25	3.29
0.125	89,000	4.25	4.16
0.125	182,000	4.25	4.22
0.125	308,000	4.25	4.28
0.375	65,000	6.41	6.45
0.375	133,000	6.41	6.48
0.375	216,000	6.41	6.51
0.562	46,000	7.73	7.69
0.562	93,000	7.73	7.69
0.562	146,000	7.73	7.70

NOTATION

A = constant in Equation (2)
A' = constant in Equation (1)
K = mixing length constant
N_{Re} = Reynolds number
r = radial distance
U^+ = dimensionless velocity
U_τ = friction velocity
y^+ = dimensionless distance from wall
β = $(U_{\tau 1}/U_{\tau 2})^2$

Subscripts

1 = inner wall region
2 = outer wall region
m = position of maximum velocity

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A Penetration Theory Model for Vapor-Phase Mass Transfer on Distillation Trays

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The University of Delaware Final Report of the AIChE Research Committee (*1*) summarizes mass transfer relationships for distillation trays which lead to the equation for the vapor phase:

$$N_G = 2a\left(\frac{D_G}{\pi t'_G}\right)^{0.5}\left(\frac{Z_f}{Z_f - Z_c}\right) t_G \qquad (1)$$

Equation (1) includes the penetration theory model for the mass transfer coefficient. Hughmark (*2*) assumed that the vapor residence time t_G is the plug flow residence time

$$t_G = \frac{Z_f - Z_c}{u_G} \qquad (2)$$

and that only one surface renewal occurred during this

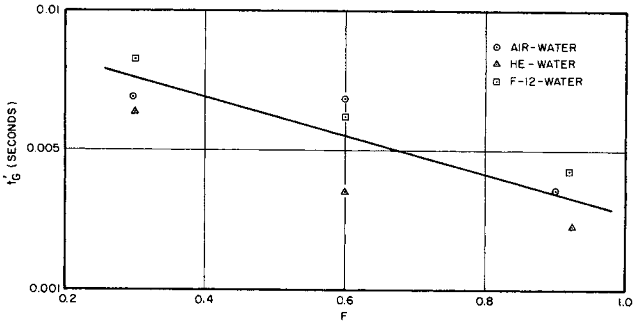

Fig. 1. Gas-phase eddy contact time.

contact time. Thus, the vapor residence time t_G and the eddy contact time t'_G were assumed to be equal.

Porter, King, and Varshney (*3*) have used carbon dioxide absorption in aqueous sodium hydroxide solution to determine the interfacial area for a range of gas flows on a 3-ft. diameter bubble cap tray. These data were obtained with a 1½-in. weir and a liquid flow of about 7 gal./min./ft. of weir. Humidification data were obtained for similar conditions at the University of Michigan as part of the AIChE program (*4*). Data are available for air-water, helium-water, and Freon 12-water for a bubble cap tray with a 1½-in. weir and a liquid flow of 12.8 gal./min/ft. Data for ammonia absorption from air with a 2-in. weir and a range of liquid flow rates were also obtained on the Michigan tray. Thus, the humidification data for a 1½-in. weir can be estimated to correspond to the liquid rate used by Porter et al. Froth and clear liquid heights are also reported for the Michigan data. Combination of the Porter et al. data for interfacial area with the Michigan humidification data, which represent total vapor-phase resistance, can then be used to estimate the eddy contact time from Equation (1). Figure 1 shows the eddy contact time t'_G as a function of F factor. It is observed that the three sets of data are reasonably consistent and that these data indicate that the assumption of one surface renewal for the vapor phase is not valid. Approximately twenty surface renewals are represented by the eddy contact times shown by Figure 1.

It is interesting to consider the eddy contact times for the vapor phase with respect to momentum transfer from the vapor to the liquid. The concept of momentum transfer from a fluid to a rigid interface has been shown by Hanratty (*5*). The instantaneous rate of momentum transfer with the liquid for a gas or vapor eddy which has been in contact with the liquid for a time t is

$$\tau_0 g_c = \rho u_c \sqrt{\frac{\mu}{\rho \pi t}} \tag{3}$$

The Danckwerts modification of the penetration theory provides a range of values of t with the probability for a gas element to have been in contact with the liquid for a time between t and $t + dt$:

$$\phi(t)\,dt = \frac{dt}{t'_G} \tag{4}$$

Averaging the effect of all elements in contact with the liquid, we get

$$\tau_0 g_c = \int_0^{t'_G} \rho u_c \sqrt{\frac{\mu}{\rho \pi t}}\ \frac{dt}{dt'_G} = 2\rho u_c \sqrt{\frac{\mu}{\rho \pi t'_G}} \tag{5}$$

The momentum for a distillation tray is represented by ρu_G^2 and $F = u_G\sqrt{\rho}$, so Equation (5) becomes

$$F = 2\,\frac{u_c}{\sqrt{\tau_0 g_c/\rho}}\sqrt{\frac{\mu}{\pi t'_G}} \tag{6}$$

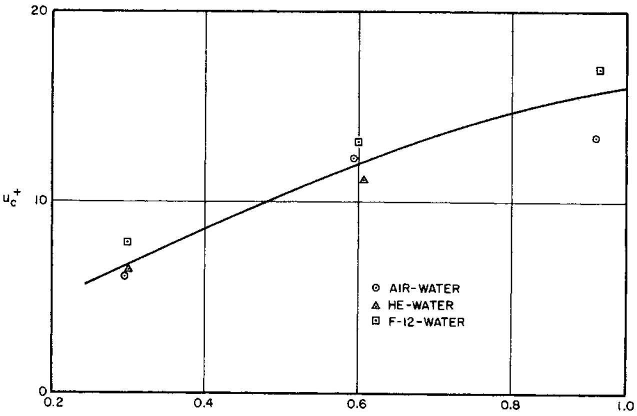

Fig. 2. Dimensionless gas-phase velocity.

and from fluid mechanics

$$u_c^+ = u_c/\sqrt{\tau_0 g_c/\rho} \tag{7}$$

Combining Equations (6) and (7), we obtain

$$u_c^+ = \frac{F}{2}\sqrt{\frac{\pi t'_G}{\mu}} \tag{8}$$

Studies of flow in conduits indicate a mean value of 13.8 for u_c^+ with a maximum of about 20 (*6*). Equation (8) has been used to estimate u_c^+ for the data shown by Figure 1. Figure 2 shows these calculated data as a function of the F factor. The results are interesting because u_c^+ could be expected to be zero at $F = 0$ for this system, and u_c^+ could be expected to approach a value of approximately 20 at high F. This system represents a free interface in comparison to the rigid interface in pipe flow; therefore, the maximum value of u_c^+ could be expected.

NOTATION

a = interfacial area per unit of total holdup
D_G = vapor-phase molecular diffusivity
F = F factor, $u_G\sqrt{\rho}$
g_c = conversion factor
N_G = number of vapor-phase mass transfer units
t = time
t_G = vapor residence time
t'_G = total time for which a gas or vapor mass has been in contact with the liquid interface
u_c = average velocity of masses of fluid exchanging momentum with the liquid interface
u_G = vapor velocity based upon tray bubbling area
Z_c = clear liquid height
Z_f = froth height
μ = vapor viscosity
ρ = vapor density
τ_0 = shear stress at liquid interface

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